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# LETTER TO THE EDITORS

## EFFECT OF OPERATING CONDITIONS, PHYSICAL SIZE AND FLUID CHARACTERISTICS ON THE GAS SEPARATION PERFORMANCE OF A LINDERSTRØM-LANG VORTEX TUBE

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THE WORK by Marshall [1], contains a number of interesting results, which should prove useful for the understanding of gas separation in the vortex tube. In particular, the results may be used for testing previously established theories [2–7] on the functioning of tubes of the design employed in [1].

According to the theoretical model considered in these studies, the main region of separation is found at intermediate radii. There, maximum tangential velocity is reached because, in tubes of the design employed, angular momentum is carried far towards the centre in a comparatively strong radial flow of gas. Furthermore, the model explains the complex relation found in both [1] and [2] between separation effect $\alpha_{pf}-1$ and cut, $\theta$, as resulting from a delicate balancing of contributions to the separation from different parts of a complex counter-current axial flow system, which is postulated to exist at intermediate radii in the tube.

Such an axial convective flow system has been shown experimentally to exist [8–10]. It has been explained qualitatively and, in the incompressible approximation, quantitatively [11] as caused by a large influx of gas in the end-wall boundary layers of the tube, a flow that necessarily must be rejected at intermediate radii. Angular momentum may be preserved to a large extent in this boundary-layer flow.

In [11] both preservation of angular momentum and diversion of flow into the boundary layers were shown to be governed by two parameters: (1) $A$, essentially a swirl parameter, $v_p r_p^2/(F/\rho)$, where $2\pi F/\rho$ is total mean volume flow through the tube, with $\rho$ a mean density, and where $v_p$ and $r_p$ are peripheral tangential velocity and radius, respectively; and (2) $Re_r$, a radial Reynolds number, usually turbulent, $=F/\rho\varepsilon L$, where $\varepsilon$ is turbulent diffusivity for momentum and $L$ is tube length. Compressibility effects appear only to have a limited influence on these results [11].

It has been found [11–13] that the parameter $\rho\varepsilon/\mu$, where $\mu$ is the viscosity, may be expressed approximately as

$$\rho\varepsilon/\mu = C(Re_{t,p})^f \tag{1}$$

where $Re_{t,p} = \rho_p v_p 2r_p/\mu$, is the tangential Reynolds number, and where $f$ and $C$ are constants with $f$ about unity or 0.8.

Furthermore, experience shows, that a velocity reduction $v_p/v_j$ takes place on entrance of the gas into the tube [14, 15]. This reduction appears to be primarily a function of the ratio of tube wall area to area of jet cross section. According to this rule as well as directly from pressure measurements in [3], $v_p/v_j$ is equal to about 1/5 in the tubes considered here.

Reference [1] extends previous measurements in [2] to wider ranges of gases, pressures and tube dimensions. The resulting range of $A$ and $Re_r$ covered is, however, no greater than in all cases a significant diversion of flow into the boundary layers is to be expected according to the theory in [11]. Thus it should be a reasonable procedure to view all the results in [1] on a common basis.

Isotope separation theory [16] predicts that

$$\left(\frac{1}{2}\frac{\theta}{1-\theta}\right)^{1/2}\cdot(\alpha_{pf}-1) = \left(\frac{\Delta U}{2\pi F}\right)^{1/2} \tag{2}$$

where $\Delta U$, the so-called separative power, is a quantity based on the diffusion equation. Its magnitude may be estimated once the flow pattern is completely known. $\Delta U$ takes into account the effect of the interplay between rate of diffusion and throughput on creating a net concentration difference between two outgoing streams.

As the axial flow pattern normally is poorly known, a direct calculation is very inaccurate [4]. A simpler and quite instructive procedure, and the one to be followed below, is to calculate the maximum possible separation regardless of axial and radial flow pattern and to assume that the ratio of actual to theoretical values is a simple function of the parameters governing this flow system. The maximum value of $\Delta U$ for a given tangential velocity distribution in the radial direction has been shown [16, 7] to be given by

$$UM \equiv \left(\frac{\Delta U_{\max}}{2\pi F}\right)^{1/2} = \frac{1}{(Re)^{1/2}}\cdot\frac{1}{(\rho\varepsilon_c/\mu)^{1/2}}\left(\frac{L}{2r_j}\right)^{1/2} Sc\frac{\Delta M}{\overline{M}}\gamma \times\left[\left(\frac{v_p}{v_j}\right)^2 M_j^2 B^{1/2}\right]. \tag{3}$$

Here $Re$, the jet Reynolds number, $=(\rho_j v_j 2\cdot r_j)/\mu$, with $2r_j$ the jet diameter (in [1], $d$); $L$ is tube length, $Sc$ is Schmidt number, $=\rho D/\mu$, and $\Delta M/\overline{M}$ is molecular weight difference divided by mean molecular weight. Furthermore, $M_j$ is jet Mach number, and

$$B \equiv \int_0^1\int_0^1\left(\frac{r_p}{v_p}\right)^4\left(\frac{v}{r}\right)^4\left(\frac{r}{r_p}\right)^2 \mathrm{d}\left(\frac{r}{r_p}\right)^2 \mathrm{d}\frac{z}{L}. \tag{4}$$

To the extent that the above boundary-layer theory applies, we may write $B = B(Re_r, A)$. The underlying diffusion equation is derived on the assumption [17, 14] that the diffusion flux of gas driven by the concentration gradients is determined by the turbulent diffusivity, while the pressure diffusion due to the rotational motion has its normal laminar value. $\rho\varepsilon_c/\mu$ may differ from $\rho\varepsilon/\mu$, because the turbulence level may decrease towards the tube axis [9, 7].

The tangential velocity, $v$, may be approximated [12] by a forced vortex, $v/r =$ a constant, in the centre region at radii less than a characteristic radius, $r^*$; and by $v \propto 1/r^n$, where in the present cases $0 \lesssim n \leqslant +1$, between $r^*$ and $r_p$. The interplay between radial inflow, peak tangential velocity and radial pressure gradient imposes a lower limit on $r^*$, at perhaps 2/3 of the (characteristic) exit orifice radius. The reason is that the radial distribution of axial pressure gradients gives rise to an outflow through the exit orifice in an annulus near the periphery, and that any tendency to distribute the flow more evenly would be opposed by the increased radial pressure drop associated with such change. In fact both experimental and theoretical evidence suggest that tangential Mach numbers above 1.2 do not obtain [15].

This conclusion and the fact that sonic conditions, except at the lowest $p_0/p_e$, exist in the inlet jet, i.e. $M_j = 1$, together with the assumption that $v_p/v_j = 0.2$, indicate that the maximum of $(v_p/v_j)^2 M_j^2 B^{1/2}$ is about unity, a limit which according to the boundary-layer theory obtains at large $Re_r$ and/or $1/A$ values.

In [1], a conspicuous feature of the results is the finding (Fig. 4) that the separation effect ($\alpha_{pf}-1$) has a maximum value when plotted against $Re$ (which is changed by varying $p_0$).

Qualitatively this is what (3) predicts should happen. Increasing $Re$ (at constant $p_e$) implies increasing rate of mass flow, and at the same time increased volume flow through the tube, because the pressure level inside the tube is only slowly rising, since it is essentially tied to $p_e$ (unless choking in the exits occurs, see below). Thus, both $Re_r$ and $1/A$ increase with $Re$, and $B$ will therefore approach its maximum value. The maximum of $UM$ (equation 3) is reached when $B$ increases at the same rate as does $Re$, or rather $Re \cdot \rho\varepsilon/\mu$ since $Re_{t,p}$ (1) slowly changes with $p_p$.

As a detailed analysis reveals, a contributing factor to the low experimental results at high $Re$ may be found in the reduction that the experimental limit to be obtained relative to $UM$ suffers at high enough $Re$-values. This reduction takes place because, with increasing throughput, the time the gas spends in the tube becomes too short for the diffusive motions to have an optimum effect on the separation.

In [3], a case was considered using air in a tube of standard size (as defined in [1]) and $p_0/p_e = 4.5$, in which the flow must have been almost identical to that in the standard tube experiments in [1] with the gas mixture $N_2/N_2O$ and $Re = 60000$. In [3] pressure measurements indicated that $n = +\frac{1}{2}$; this value was in qualitative agreement with the prediction of the boundary-layer theory as regards preservation of angular momentum. With $M_j = 1$ and $v_p/v_j = 0.2$, it follows that $(v_p/v_j)^2 M_j^2\, B^{1/2} \simeq 0.2$. Furthermore, in [7], a comparison between mass separation and energy separation in the vortex tube is made, and it is argued that the turbulent diffusivity necessarily must decrease towards the axis, or the two sets of results would be difficult to reconcile. In one case in [7], where both effects followed the same kind of pattern as functions of flow fraction, $\theta$, the results indicated that the turbulent diffusivity in the active separation region was reduced to 1/6 of the magnitude determined directly from the $v \propto 1/r^n$ dependence or indirectly from (1). With this result, $UM = 4.1 \times 10^{-3}$, where the best experimental value in [1] is $2.6 \times 10^{-3}$.

Another very interesting feature in Fig. 4 of [1] is the finding that maximum separation occurs at $Re$ values of about 60000 (when the exit pressure is 1 atm) regardless of tube size (scaling all dimensions a factor of 2.57 up or 2.0 down) and of gas properties (changing from nitrogen to helium).

This result does not follow directly from the theory described above. A detailed analysis rather suggests that $p_0/p_e$ ($p_e$ constant) should be quite a powerful parameter for unifying all such data.

In fact, there is little doubt that this is the case as regards the variation of $B = B(Re_r, A)$, based as this is on a considerable amount of experimental evidence. Therefore, the explanation for the discrepancy most probably lies with the limited efficiency, with which the axial convective system is able to produce a net separation effect. Qualitatively, the cause may be the fact that the change due to scaling up or down or to change of gas does not affect $Re_r$ and $A$ at a given $p_0/p_e$ in quite the same way, so that a pressure adjustment cannot bring both parameters back to their previous magnitudes simultaneously. The convective flow system may well be much more sensitive to this kind of dissimilarity than the tangential velocity distribution (and $B$). Alternatively, the model may be too primitive to serve as a reliable guide in this case.

The separation, being a diffusive process, must have a maximum independent of throughput, when its efficiency is measured in terms of amount transferred rather than concentration difference (except in so far as the throughput affects the turbulent diffusivity). $\Delta U_{\max}$ [or $UM(2\pi F)^{1/2}$, conveniently expressed as $g^{1/2}\,s^{-1/2}$, see (2)] is therefore for some purposes a more useful parameter than $UM$. In Fig. 1,

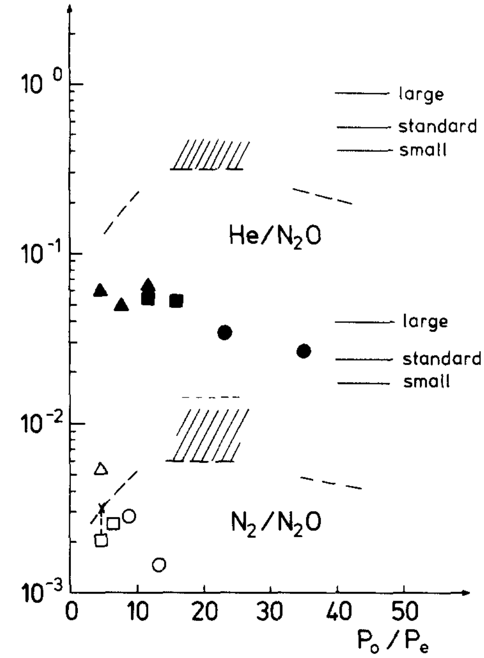


FIG. 1. Experimental $[\frac{1}{2}(\theta/1-\theta)]^{1/2}(2\pi F)^{1/2}(\alpha_{pf}-1)$ values plotted against overall pressure ratio, $p_0/p_e$; ○, ● small, □, ■ standard, and △, ▲ large tubes. $p_e = 1$ atm; $Re \geqslant 60000$. Dashed lines and ×, corresponding calculated turbulent $UM(2\pi F)^{1/2}[=(\Delta U_{\max})^{1/2}]$ values. Solid lines, calculated laminar upper limits to $UM(2\pi F)^{1/2}$.

calculated values of maximum $UM(2\pi F)^{1/2}$ are indicated, together with the corresponding laminar values, and experimental points from [1] are plotted against $p_0/p_e$. Only measurements where $Re > 60000$ are shown, the purpose being to test how closely experiment approaches the upper limit of $UM(2\pi F)^{1/2}$. The range indicated by shading represents the effect of $\varepsilon_c < \varepsilon$.

The theoretical point at 4.64 atm. represents the calculated value for $N_2$-gas, referred to above. The conjectured change in $UM(2\pi F)^{1/2}$, when $B$ approaches its maximum value, is also indicated.

At $p_0/p_e > 25$, choking in the exits is bound to occur. This follows from the fact that the ratio of exit area to jet area is 13 and that, with choking in both inlet and exits, $\rho_0/\rho_j \simeq 1.54$ and $\rho_{\text{external}}/\rho_{\text{exit}}, \simeq 2/1.54$, so that $\rho_0/\rho_{\text{external}} = p_0/p_e \simeq 25$. In fact choking probably happens already at somewhat smaller $p_0/p_e$, because swirl in an exit reduces the outflow through it. The possible effect of choking on $UM(2\pi F)^{1/2}$ through an increase of $Re_{t,p}$ is also indicated in the figure.

It is seen, that the experimental separation maxima for both gases are of the order 10% of the theoretical laminar values. Apparently the lower turbulence level in the He-tubes ($Re_{t,p}$ smaller) does not bring about better separation. The reason may conceivably be either that the throughput in all cases is so large that turbulent back-diffusion, which is proportional to the slope of the concentration gradients established, plays a minor part, or alternatively that the convective flow system in the He-case never reaches as efficient a state as with $N_2$.

The result in Fig. 5 of [1] that $\Delta M/\bar{M}$ correlates data for different gases at constant $Re$ is a corollary of the above result.

In one set of measurements in [1] (Fig. 6), starting from the standard conditions considered in detail above ($N_2$-gas in standard tube, $Re = 60000$), $p_e$ was reduced keeping $p_0$ constant. This change reduces both $A$ and the turbulent diffusivity, and $Re_r$ is increased. Thus $B$ approaches its maximum value. In Table 1$a$, experiment and theory are

Table 1.

| | | a | | b | |
|---|---|---|---|---|---|
| | normal | reduced $p$ | ratio | reduced $p$ | ratio |
| $\alpha_{pf}-1$ | $4.3\times10^{-3}$ | $9.0\times10^{-3}$ | 2.1 | $4.3\times10^{-3}$ | 3.2 |
| $p_0$ | 4.64 | 4.64 | | 1.41 | |
| $p_e$ | 1 | 0.25 | | 0.25 | |
| $p_p$ (estimated) | 1.5 | 0.75 | | 0.5 | |
| $\left(\frac{\Delta U_{\max}}{2\pi F}\right)^{1/2}$ | | | $1.41\,\frac{(B_{\mathrm{red}})}{(B_{\mathrm{norm}})}$ | | 3.2 |

compared. $(B_{\mathrm{red}}/B_{\mathrm{norm}})^{1/2}$ may readily account for the remaining factor of 1.5, and thus qualitatively the increased separation effect is explained.

By further reducing $p_e$, i.e. at $p_0/p_e$-values above approximately 30, the separation effect was found in [1] to decrease somewhat again. Against this, the model predicts that choking in the exits above $p_0/p_e \simeq 25$ should lead to a constant separation effect independent of the external pressure. The explanation for this discrepancy is probably that the outflow through the exits is not choked over the entire cross-section at once. Since reversed flow along the axis without doubt exists especially in the critical pressure region, changes in the axial convective system may take place while the flow in the exits becomes completely choked, and this may give rise to the reduction in $\alpha_{pf}-1$ observed.

In another set of experiments in [1] both $p_0$ and $p_e$ were reduced. This changes the turbulence level, but does not influence $Re_r$ much, nor is $A$ changed appreciably; thus $B$ may remain unaltered. In Table 1b, experiment and theory are compared. Although the close agreement is entirely fortuitous, the tendency is probably real enough.

There is a limit to how large $UM$ may become by reducing the pressure level (at constant $p_0/p_e$). Thus, when laminar conditions are reached, further reduction in pressure will affect $Re_r$ and change the tangential velocity distribution in such a way that $B$ decreases. In addition, at small enough throughputs, the obtainable experimental limit relative to $UM$ decreases, because the equilibrium concentration gradient is so readily established within the tube, that a decreasing part of the tube volume participates in the separation process.

In [1], Fig. 7 almost all data for $Re < 60\,000$ are plotted against the parameter $(\Delta M/\overline{M})\gamma Sc(p_0/p_e)$ and a power law for the standard tube is deduced. In view of the above discussion, both as regards the form of $UM$ and concerning the correlations established in Figs. 4 and 5 of [1], one might be led to believe that a more rewarding procedure would be to divide $\alpha_{pf}-1$ by $\Delta M/\overline{M}\gamma Sc$ and to plot the remaining quantity against $Re$ (with $p_e = 1$ atm). Since most data are for $Re = 60\,000$, the result is mainly a contracted view of Fig. 5 of [1]; however, the tail of results going to small $Re$-values does indicate, that the dependence of the "normalised" separation effect on $Re$ may be linear. On the other hand, with the data plotted in this way it also becomes obvious, that neither the influence of tube dimensions nor of gas properties is entirely accounted for by this method. Thus, the relationship established in [1] may be the best to be had at present.

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